

ШИФР 2934

Класс 10 Вариант 7 Дата Олимпиады 11.02.2017

Площадка написания МГТУ им. Баумана

Задача	1	2	3	4	5	6	7	8	9	10	Σ		Подпись
											Цифрой	Прописью	
Оценка	5	4	5	5	5	10	—	8	—	10	52	пятьдесят два	

$$\sqrt{1} \quad (x-1)(x-3)(x-5) = x(x^2-9)$$

$$(x^2-4x+3)(x-5) = x^3-9x$$

$$\cancel{x^3} - 5x^2 - 4x^2 + 20x + 3x - 15 = \cancel{x^3} - 9x$$

$$-9x^2 + 32x - 15 = 0$$

$$9x^2 - 32x + 15 = 0$$

$$D = 1024 - 540 = 484$$

$$x_{1,2} = \frac{32 \pm 22}{18} = \frac{3}{9} \text{ и } \frac{5}{9}$$

Ответ: $\frac{5}{9}; 3$ ✓

$$\sqrt{2} \quad \sqrt{x-1} + \sqrt{3x+1} = 2$$

$$\sqrt{x-1} = 2 - \sqrt{3x+1}$$

$$x-1 = 4 - 4\sqrt{3x+1} + 3x+1$$

$$4\sqrt{3x+1} = 2x+6$$

$$2\sqrt{3x+1} = x+3 \quad \rightarrow \geq 0?$$

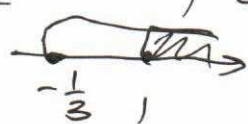
$$12x+4 = x^2+6x+9$$

$$x^2-6x+5=0 \quad \checkmark$$

$$x = 5; 1$$

Ответ: 1; 5

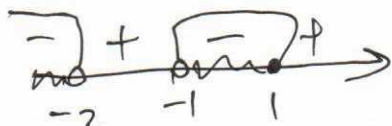
$$\text{003: } \begin{cases} x-1 \geq 0 \\ 3x+1 \geq 0 \end{cases} \quad \begin{cases} x \geq 1 \\ x \geq -\frac{1}{3} \end{cases}$$

 $[-\frac{1}{3}, 1] \rightarrow [1; +\infty)$ ✓

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№3 $\frac{2}{x+1} \geq \frac{3}{x+2}$

$$\frac{2x+4-3x-3}{(x+1)(x+2)} \geq 0 \quad ; \quad \frac{1-x}{(x+1)(x+2)} \geq 0 \quad ; \quad \frac{x-1}{(x+1)(x+2)} \leq 0$$



Ответ: $(-\infty; -2); (-1; 1)$

№4 $\log_3 x + \log_9 x + \log_{27} x = \frac{11}{12}$

ОДЗ: $x > 0$

$$\log_3 x + \frac{1}{2} \log_3 x + \frac{1}{3} \log_3 x = \frac{11}{12}$$

$$\frac{11}{6} \log_3 x = \frac{11}{12}$$

$$\log_3 x = \frac{1}{2} \quad ; \quad x = \sqrt{3}$$

Ответ: $\sqrt{3}$

№5 $8 \cdot 4^x + 1 \leq 6 \cdot 2^x$

$2^x = t, t > 0$

$$8t^2 + 1 \leq 6t$$

$$8t^2 - 6t + 1 \leq 0$$

$$D = 36 - 32 = 4$$

$$t = \frac{6 \pm 2}{16} \geq 0,5$$

$$0,5 \leq t \leq 0,25 \quad ; \quad 0,5 \leq 2^x \leq 0,25$$

$$\left\{ \begin{array}{l} 2^x \geq \frac{1}{2} \\ 2^x \leq \frac{1}{4} \end{array} \right. ; \left\{ \begin{array}{l} 2^x \geq 2^{-1} \\ 2^x \leq 2^{-2} \end{array} \right. ; \left\{ \begin{array}{l} x \geq -1, \text{ так } 2 > 1 \\ x \leq -2, \text{ так } 2 > 1 \end{array} \right.$$

Ответ: $(-\infty; -2]; [-1; +\infty)$

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Пусть a - сибирск; b - ашор; c - персид; d - славянск.,
тогда

$$a = c - 13 = 1,5d - 13 = 3b - 13; \quad c = 1,5d = 3b; \quad d = 2b$$

$$96 = 90 \quad ; \quad b = 10 \Rightarrow a = 17, c = 30, d = 20$$

Проведем см-выс в точку касания
окр

Пусть σ_K — центр S — наименьший,
 σ_B — средняя и σ_A — наибольший.

$$\Rightarrow CM = CE = X, EB = EH = 4; \checkmark$$

$$BC^2 = KB \cdot AB$$

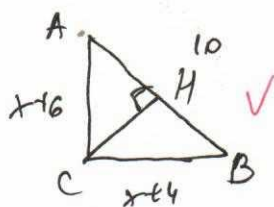
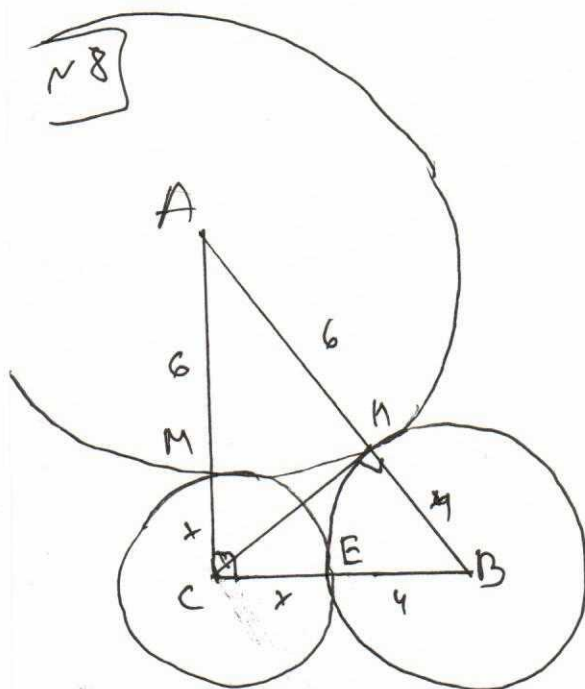
$$x^2 + 8x + 16 = 40 \quad ; \quad x^2 + 8x - 24 = 0$$

$$D = 64 + 96 = 160$$

$$x_1 = \frac{-8 + 4\sqrt{10}}{2} = -4 + 2\sqrt{10} \approx 2$$

$$x_2 = \frac{-8 - 4\sqrt{10}}{2} = \text{hegggar}$$

Omben: $-4 + 2\sqrt{10}$





ОТРАСЛЕВАЯ
ОЛИМПИАДА
ШКОЛЬНИКОВ

$(ab)c = a(bc)$ $E = mc^2$ C1=CC=CC=C1

Использовать только эту сторону листа,
обратная сторона не проверяется!

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$\sqrt[3]{10} \quad A^3 + 3A$, если $\sqrt[3]{5+2} \quad \sqrt[3]{5-2}$

$$\begin{aligned} A(A^3 + 3) &= \left((\sqrt{5+2})^{\frac{1}{3}} - (\sqrt{5-2})^{\frac{1}{3}} \right) \left((\sqrt{5+2})^{\frac{2}{3}} - 2(\sqrt{5+2})^{\frac{1}{3}}(\sqrt{5-2})^{\frac{1}{3}} \right. \\ &+ \left. (\sqrt{5-2})^{\frac{2}{3}} + 3 \right) = \sqrt{5+2} - 2(\sqrt{5+2})^{\frac{2}{3}}(\sqrt{5-2})^{\frac{1}{3}} + (\sqrt{5+2})^{\frac{1}{3}}(\sqrt{5-2})^{\frac{2}{3}} \\ &+ 3(\sqrt{5+2})^{\frac{1}{3}} - (\sqrt{5+2})^{\frac{2}{3}}(\sqrt{5-2})^{\frac{1}{3}} + 2(\sqrt{5+2})^{\frac{1}{3}}(\sqrt{5-2})^{\frac{2}{3}} - \sqrt{5+2} \\ &- 3(\sqrt{5-2})^{\frac{1}{3}} \quad \left\{ 4 + (-3(\sqrt{5+2})^{\frac{2}{3}}(\sqrt{5-2})^{\frac{1}{3}}) + 3(\sqrt{5+2})^{\frac{1}{3}}(\sqrt{5-2})^{\frac{2}{3}} \right. \\ &+ 3(\sqrt{5+2})^{\frac{1}{3}} - 3(\sqrt{5-2})^{\frac{1}{3}} \left. \right\} = 4 + (-3(\sqrt{5+2})^{\frac{1}{3}}((\sqrt{5-2})^{\frac{1}{3}} - 1) \\ &+ 3(\sqrt{5-2})^{\frac{1}{3}}((\sqrt{5+2})^{\frac{1}{3}} - 1)) = \end{aligned}$$

$$\begin{aligned} &= 4 + (-2(\sqrt{5+2})^{\frac{1}{3}}(\sqrt{5-2})^{\frac{1}{3}}(\sqrt{5+2} - \sqrt{5-2}) + (\sqrt{5-2})^{\frac{1}{3}}(\sqrt{5+2})^{\frac{1}{3}} \\ &(\sqrt{5-2} - \sqrt{5-2})) = 4 + (-8(\sqrt{5+2})^{\frac{1}{3}}(\sqrt{5-2})^{\frac{1}{3}} - 4(\sqrt{5-2})^{\frac{1}{3}}(\sqrt{5+2})^{\frac{1}{3}} \\ &= 4 - 12(\sqrt{5+2})^{\frac{1}{3}}(\sqrt{5-2})^{\frac{1}{3}} = 4(1 - 3\sqrt[3]{5-4}) = \\ &= 4 \cdot (1 - 3) = \underline{-8} \end{aligned}$$

Ответ: -8